Muddy City Problem

Description:
Once upon a time there was a city that had no roads. Getting around the city was particularly difficult after rainstorms because the ground became very muddy—cars got stuck in the mud and people got their boots dirty. The mayor of the city decided that some of the streets must be paved, but didn’t want to spend more money than necessary because the city also wanted to build a swimming pool. The mayor therefore specified two conditions:

1. Enough streets must be paved so that it is possible for everyone to travel from their house to anyone else’s house only along paved roads, and
2. The paving should cost as little as possible. Here is the layout of the city. The number of paving stones between each house represents the cost of paving that route.

Find the best route that connects all the houses, but uses as few counters (paving stones) as possible. What strategies did you use to solve the problem?

Note: The bridge is already paved.

Solutions:
The solution for this map is 23. There are several minimum spanning trees to get this optimal answer.

One possible answer is as below:
Please refer to the resource HERE if you want to know the detail of the algorithm to solve the problem.

**Kruskal's algorithm**

One good strategy to find the best solution is to start with an empty map, and gradually add counters until all of the houses are linked, adding the paths in increasing order of length, but not linking houses that are already linked. Different solutions are found if you change the order in which paths of the same length are added. Two possible solutions are shown below.

**Prim's algorithm**

Prim’s algorithm works by attaching a new edge to a single growing tree at each step: Start with any vertex as a single-vertex tree; then add V-1 edges to it, always taking next (coloring black) the minimum-weight edge that connects a vertex on the tree to a vertex not yet on the tree (a crossing edge for the cut defined by tree vertices).

### Dominating Set Problem

**Description:**

Shown on the Ice Cream Vans worksheet is a map of Tourist Town. The lines are streets and the dots are street corners. The town lies in a very hot country, and in the summer season ice-cream vans park at street corners and sell ice-creams to tourists. We want to place the vans so that anyone can reach one by walking to the end of their street and then at most one block further. *(It may be easier to imagine people living at the intersections rather than along the streets; then they must be able to get ice-cream by walking at most one block.)* The question is, how many vans are needed and on which intersections should they be placed?

**Solution:**

*Ice-Cream Town* (Minimum Covering Set)

The trick to build this problem like the two graph below. Since we want the answer to be six. Then draw six trees first, with one root and several children. Then, link the trees together. Remember, ONLY link the children together to confuse the players, do not link the root with other children.